

2.1 - Solution Curves Without a Solution

Definition: If $y = y(x)$ is a differentiable function, then $\frac{dy}{dx}$ gives the slope of a tangent line at a point. We can learn about a DE $\frac{dy}{dx} = f(x, y)$ using segments of tangent lines, called **lineal elements**. The collection of these lineal elements is a **direction field** or **slope field**.

Example: Sketch an approximate solution curve that passes through each of the indicated points

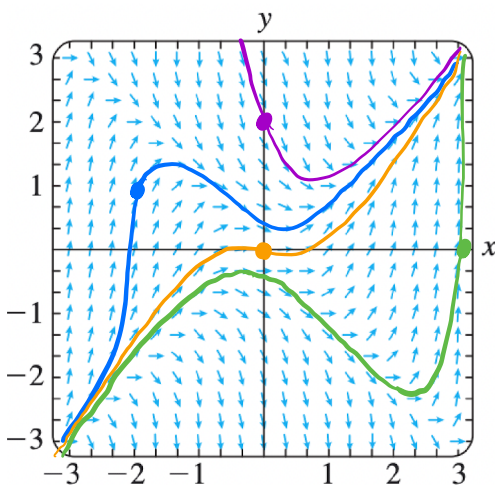
$$\frac{dy}{dx} = x^2 - y^2$$

a) $y(-2) = 1$

b) $y(3) = 0$

c) $y(0) = 2$

d) $y(0) = 0$

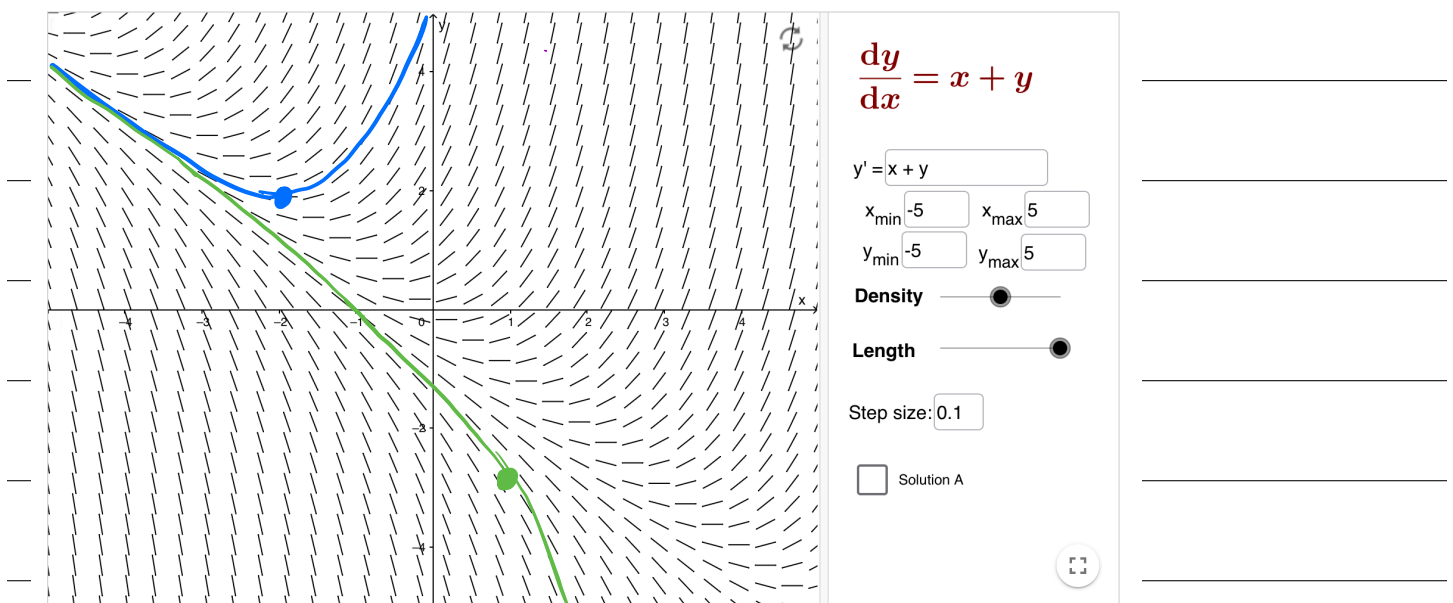


Example: Use computer software to obtain a direction field for the differential equation. Sketch an approximate solution curve passing through each of the given points.

$$y' = x + y$$

a) $y(-2) = 2$

b) $y(1) = -3$



Definition: A DE in which the independent variable does not appear explicitly is an **autonomous** differential equation. A first-order autonomous DE has the form $\frac{dy}{dx} = f(y)$.

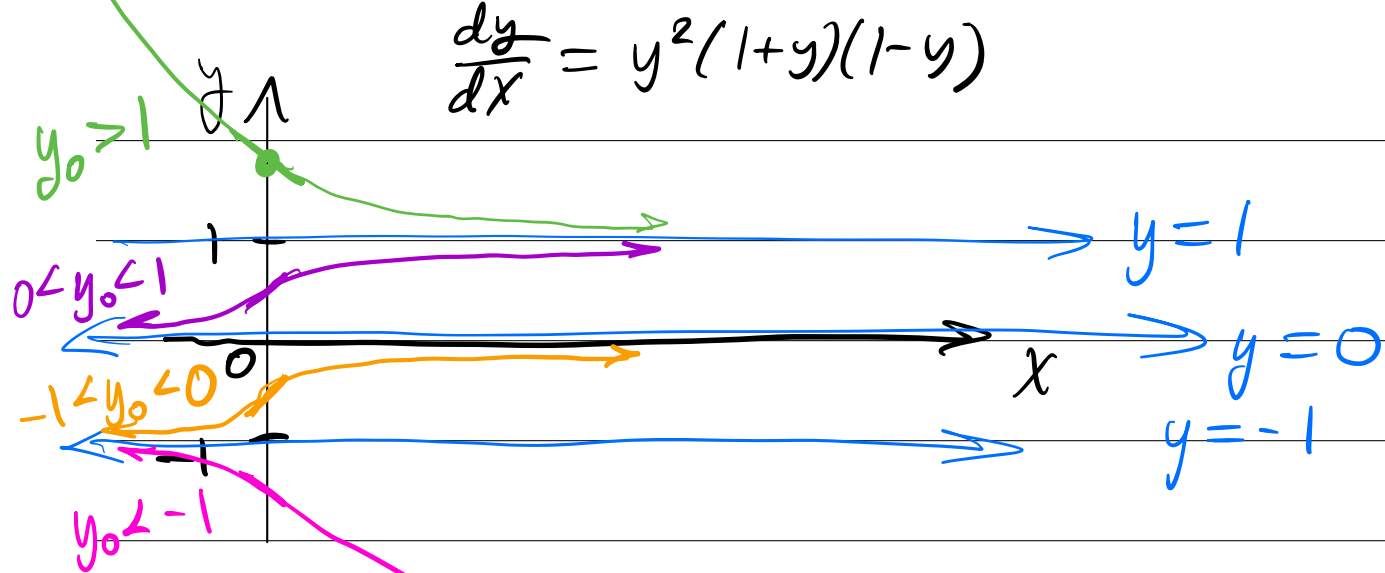
Definition: A number c is a **critical point** of an autonomous DE if $f(c) = 0$. A critical point is also known as an **equilibrium point** or **stationary point**. A constant solution $y(x) = c$ of an autonomous DE is an **equilibrium solution**.

Example: Consider the autonomous first-order differential equation $\frac{dy}{dx} = y^2 - y^4$ and the initial condition $y(0) = y_0$. Sketch the graph of a typical solution $y(x)$ when y_0 has the given values.

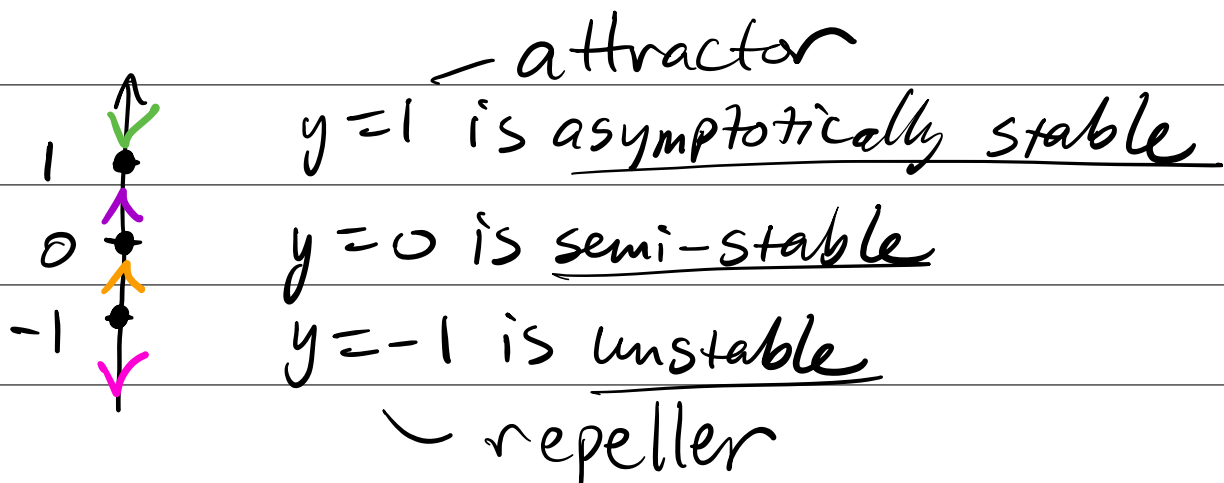
- a) $y_0 > 1$ b) $0 < y_0 < 1$ c) $-1 < y_0 < 0$ d) $y_0 < -1$

$$\frac{dy}{dx} = y^2(1-y^2) \Rightarrow \frac{dy}{dx} = y^2(1+y)(1-y)$$

$$\frac{dy}{dx} = 0 \text{ if } y = \underline{-1}, \underline{0}, \underline{1}$$



The above information can be collapsed into a 1-D graph, called a phase portrait:



No solution curve will cross a horizontal asymptote associated with a critical point.
 (uniqueness of solution thm)

